L8 拓展篇第 6 讲—二次根式

参考答案与试题解析

1.用待定系数法化简：(1)$\sqrt{4+2\sqrt{3}}$；(2)$\sqrt{7-2\sqrt{10}}$.

【答案】(1)$1+\sqrt{3}$；(2)$\sqrt{5}-\sqrt{2}$.

2.化简：$\sqrt{13+4\sqrt{2}+4\sqrt{7}+2\sqrt{14}}$.

【答案】$2+\sqrt{2}+\sqrt{7}$.

3.化简：$\sqrt{9+\sqrt{53+8\sqrt{6}}}+\sqrt{9-\sqrt{53+8\sqrt{6}}}$.

【答案】$2\sqrt{3}+\sqrt{2}$

【分析】设 $m=\sqrt{9+\sqrt{53+8\sqrt{6}}}+\sqrt{9-\sqrt{53+8\sqrt{6}}}$ ，则

$m^2=9+\sqrt{53+8\sqrt{6}}+9-\sqrt{53+8\sqrt{6}}+2\sqrt{9^2-\left(53+8\sqrt{6}\right)}$ ，

$m^2=18+2\sqrt{28-8\sqrt{6}}=18+2\sqrt{\left(2\sqrt{6}\right)^2+2^2-2\cdot2\sqrt{6}\cdot2}=18+2\sqrt{\left(2\sqrt{6}-2\right)^2}$

$=18+2\left(2\sqrt{6}-2\right)=14+4\sqrt{6}=\left(2\sqrt{3}\right)^2+\left(\sqrt{2}\right)^2+2\cdot2\sqrt{3}\cdot\sqrt{2}=\left(2\sqrt{3}+\sqrt{2}\right)^2$ ，

即 $m=2\sqrt{3}+\sqrt{2}$. 所以 $9+\sqrt{53+8\sqrt{6}}+\sqrt{9-\sqrt{53+8\sqrt{3}}}=2\sqrt{3}+\sqrt{2}$.

4.化简 $S=\sqrt{x+2\sqrt{x-1}}+\sqrt{x-2\sqrt{x-1}}\,(x>1)$.

【答案】$S=\begin{cases}2\sqrt{x-1}, & x\geqslant 2,\\ 2, & 1<x<2\end{cases}$.

【分析】由题意：$S^2=2x+2|x-2|$，

所以$S=\begin{cases}2\sqrt{x-1}, & x\geqslant 2,\\ 2, & 1<x<2\end{cases}$.

5.化简 $\sqrt{1+a^2+\sqrt{1+a^2+a^4}}$

【答案】$\frac{\sqrt{2}}{2}\left(\sqrt{a^2+a+1}+\sqrt{a^2-a+1}\right)$.

【分析】原式$=\sqrt{\frac{2+2a^2+2\sqrt{1+a^2+a^4}}{2}}=\sqrt{\frac{(a^2+a+1)+2\sqrt{(a^2+a+1)(a^2-a+1)}+(a^2-a+1)}{2}}$

$$=\sqrt{\frac{\left(\sqrt{a^2+a+1}+\sqrt{a^2-a+1}\right)^2}{2}}=\frac{\sqrt{2}}{2}\left(\sqrt{a^2+a+1}+\sqrt{a^2-a+1}\right).$$

6.若 $a=\sqrt{2}+1$，计算

$$\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cdots\cfrac{1}{2+\cfrac{1}{a}}}}}$$（共有 200 层）的值.

【答案】$\sqrt{2}-1$

【分析】因为 $a=\sqrt{2}+1$，所以 $\frac{1}{a}=\frac{1}{\sqrt{2}+1}=\sqrt{2}-1$，

所以 $2+\frac{1}{a}=\sqrt{2}-1+2=\sqrt{2}+1=a$，

所以 $\cfrac{1}{2+\cfrac{1}{a}}=\frac{1}{a}=\sqrt{2}-1$.

所以，不论多少层，原式 $=\frac{1}{a}=\sqrt{2}-1$.

7.计算：

$$S=\sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}}+\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}+\sqrt{1+\frac{1}{3^2}+\frac{1}{4^2}}+\cdots+\sqrt{1+\frac{1}{2023^2}+\frac{1}{2024^2}}.$$

【答案】$S=2023\frac{2023}{2024}$.

【分析】$\sqrt{1+\frac{1}{n^2}+\frac{1}{(n+1)^2}}=\sqrt{\left(1+\frac{1}{n}\right)^2-\frac{2}{n}+\frac{1}{(n+1)^2}}=\sqrt{\left(\frac{n+1}{n}\right)^2-2\cdot\frac{n+1}{n}\cdot\frac{1}{n+1}+\left(\frac{1}{n+1}\right)^2}$

$$=\sqrt{\left(\frac{n+1}{n}-\frac{1}{n+1}\right)^2}=\frac{n+1}{n}-\frac{1}{n+1}=1+\frac{1}{n}-\frac{1}{n+1}.$$

所以

$$S=\left(1+\frac{1}{1}-\frac{1}{2}\right)+\left(1+\frac{1}{2}-\frac{1}{3}\right)+\left(1+\frac{1}{3}-\frac{1}{4}\right)+\cdots+\left(1+\frac{1}{2023}-\frac{1}{2024}\right)=2023+1-\frac{1}{2024}=2023\frac{2023}{2024}.$$